Prove that a matrix $A \in \mathbb{R}^{n \times n}$ with $n$ distinct eigenvalues has $n$ linearly independent vectors.
2 (15 points). *******
Prove that all eigenvalues of an Hermitian matrix are real, and that eigenvectors corresponding to distinct eigenvalues are orthogonal.
3 (20 points). *****

Compute $A^6$, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

Show that $AX = DX$ too.
4 (10 points). Show that $e^A$ is nonsingular for any diagonalizable matrix $A$
5 (10 points). Show that if $A$ is stochastic, then $\lambda = 1$ is its eigenvalue.
6 (15 points). ******
Find a unitary diagonalizing matrix for each of the following:

\[ A = \begin{pmatrix} 1 & 3 + i \\ 3 - i & 4 \end{pmatrix} \]

Is \( A \) Hermitian?
Let $U$ be a unitary matrix. Prove:

(a) $U$ is normal;

(b) $||U\mathbf{x}|| = ||\mathbf{x}||$ for any $\mathbf{x} \in \mathbb{C}^n$;

(c) if $\lambda$ is an eigenvalue of $U$, then $|\lambda| = 1$.
8 (15 points). Prove that a eigenvectors of a normal matrix $A \in C^{n \times n}$ form an orthonormal basis for $C^n$. 
